

On Low-temperature Dephasing by Electron-electron Interaction^{*}

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The quantum coherence of electrons can be probed by studying weak localization corrections to the conductivity. Interaction effects lead to dephasing, with electron-electron interactions being the important intrinsic mechanism. A controversy exists whether or not the dephasing rate, as measured in a weak localization experiment, vanishes at low temperatures. We review the non-perturbative analysis of this question and some of the arguments which have been raised against it. The compact form of the presentation should make the derivation more transparent and accessible for discussions. We also compare with recent experiments.

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1. INTRODUCTION

Quantum coherence and interference are basic principles of quantum mechanics as well as essential for the concepts of quantum state engineering. Loss of phase coherence results in a suppression of interference and signals a transition from quantum to classical behavior. Thus, it is crucial to understand how quantum coherence is destroyed in various physical systems.

Isolated quantum degrees of freedom preserve phase coherence. *Inelastic* interactions with other – quantum or classical – degrees of freedom destroy

^{*}Dedicated to P. Wölffe on the occasion of his 60th birthday.

coherence. For electrons in metals at low temperatures the dominant intrinsic interaction is electron-electron scattering. As the temperature is lowered further, these inelastic processes freeze out, and the scattering rate vanishes¹, $1/\tau_{\text{in}} \propto T^2$. Largely because of this property it is frequently assumed that also the electron decoherence rate $1/\tau_{\varphi}$ should vanish as the temperature approaches zero.

The electron decoherence can be probed by studying the weak localization correction to the conductivity. While many of the earlier experiments² appeared to confirm the theoretical expectations^{3,4,5}, more recent work^{6,7} (see Ref. 7 for further references) suggested that the electron dephasing time τ_{φ} in disordered metals saturates at sufficiently low temperatures. Extensive experimental checks⁶ allowed to rule out various sources for this saturation, such as electron scattering from magnetic impurities, heating and external high frequency noise. Several other saturation mechanisms had been explored, such as $1/f$ noise, interaction of electrons with nuclear magnetic moments or with two-level systems in the two-channel Kondo regime⁸. They appear not to play a role in the experiments described in Ref. 7, nor in the more recent studies of Ref. 9. As concluded in Ref. 7, these recent experimental findings “reinforce the earlier conclusion that the saturation is a real effect, most likely arising from electron-electron interaction”.

A low-temperature saturation of τ_{φ} due to electron-electron interactions has drastic physical consequences^{6,10}. One of them is that the Coulomb interaction precludes (strong) Anderson localization in disordered conductors. Another is that the Fermi liquid theory may not apply to disordered conductors. Dephasing at low temperatures also imposes practical limitations on quantum state engineering with electronic devices. All that makes the low-temperature electron decoherence one of the most intriguing issues of modern condensed matter physics.

In this context it is important to note that several model systems where a quantum particle is coupled to a bath (e.g. a bath of harmonic oscillators as in the Caldeira-Leggett model¹¹) yield results which indicate a nonvanishing dephasing rate even at zero temperature^{12,13,14,15}. We further like to mention the work of A.G. Aronov and P. Wölfle¹⁶ which emphasizes the fundamental importance of dephasing due to fluctuating magnetic field for such issues as high temperature superconductivity and the metal-insulator phase transition.

The aim of the present paper is to review the theoretical analysis of the Coulomb-interaction-induced quantum dephasing of electrons in disordered conductors derived before by two of the present authors (GZ)^{10,17,18}. This work has stimulated a heated discussion and exchange of lengthy publications. One of our goals is to present in a compact form the main steps of

Low-temperature Dephasing ...

the derivation, which should help an interested reader to judge the validity of this approach. Some of the more recent work – in particular we like to mention that of von Delft¹⁹ – have put the discussion on a more rational basis by explicitly pointing out approximations which had been used. The second goal of this article is to justify these approximations. Finally, we will compare with some recent experiments with the aim to judge whether they can help to discriminate between different theoretical views.

The structure of the paper is as follows: We will first review the derivation of weak localization effects on the conductivity as well as a simple analysis of the influence of electron-electron interactions. At this level we reproduce some important results, but, in addition, we introduce the formalism and notations which will be used in later sections, where the same questions will be analyzed on a more rigorous level. We then summarize in compact form the derivation which has been presented before by two of us (GZ)^{10,17,18}. This part should make explicit which approximations have been used. We then will present several arguments which had been raised against the conclusions of GZ as well as our counterarguments. The final section is devoted to a comparison with recent experiments.

2. WEAK LOCALIZATION BASICS

In this section we shortly review the derivation of the weak localization correction to the conductivity for non-interacting electrons using a path integral formulation, and we present a hand-waving approach to account for the effects of electron-electron interactions.

The density matrix of a quantum particle evolves according to

$$\rho(t_f, \mathbf{r}_{1f}, \mathbf{r}_{2f}) = \int d\mathbf{r}_{1i} d\mathbf{r}_{2i} J(t_f, t_i, \mathbf{r}_{1f}, \mathbf{r}_{2f}, \mathbf{r}_{1i}, \mathbf{r}_{2i}) \rho(t_i, \mathbf{r}_{1i}, \mathbf{r}_{2i}), \quad (1)$$

where the time evolution operator J combines the forward and backward propagators typical for the density matrix. For a single particle which moves in an impurity potential $U(\mathbf{r})$ and which is subject to a static applied electric potential V_x , this operator can be expressed as a path integral over two paths

$$\begin{aligned} J(t_f, t_i, \mathbf{r}_{1f}, \mathbf{r}_{2f}, \mathbf{r}_{1i}, \mathbf{r}_{2i}) &= \\ &= \int_{\mathbf{r}_1(t_i)=\mathbf{r}_{1i}}^{\mathbf{r}_1(t_f)=\mathbf{r}_{1f}} \mathcal{D}\mathbf{r}_1 \int_{\mathbf{r}_2(t_i)=\mathbf{r}_{2i}}^{\mathbf{r}_2(t_f)=\mathbf{r}_{2f}} \mathcal{D}\mathbf{r}_2 e^{iS_0[\mathbf{r}_1] - iS_0[\mathbf{r}_2] + i \int_{t_i}^{t_f} dt' [eV_x(\mathbf{r}_1(t')) - eV_x(\mathbf{r}_2(t'))]}, \end{aligned} \quad (2)$$

where $S_0[\mathbf{r}] = \int_{t_i}^{t_f} dt' [m\dot{\mathbf{r}}^2(t')/2 - U(\mathbf{r}(t'))]$.

D.S. Golubev, A.D. Zaikin, and G. Schön

We are interested in the conductivity in spatially homogeneous and stationary situations. We, therefore, assume $V_x(\mathbf{r}) = -\mathbf{E}\mathbf{r}$ and calculate the current at \mathbf{r} and t_f due to the field at $\mathbf{r}(t')$ and t' in an expansion of the density matrix to linear order in V_x . Thus we get

$$\sigma = \frac{e^2}{3m} \int_{t_i}^{t_f} dt' \int d\mathbf{r}_{1i} d\mathbf{r}_{2i} [\nabla_{\mathbf{r}_{1f}} - \nabla_{\mathbf{r}_{2f}}] \int_{\mathbf{r}_{1i}}^{\mathbf{r}_{1f}} \mathcal{D}\mathbf{r}_1 \int_{\mathbf{r}_{2i}}^{\mathbf{r}_{2f}} \mathcal{D}\mathbf{r}_2 [\mathbf{r}_1(t') - \mathbf{r}_2(t')] \times e^{iS_0[\mathbf{r}_1] - iS_0[\mathbf{r}_2]} \rho(t_i, \mathbf{r}_{1i}, \mathbf{r}_{2i}) \Big|_{\mathbf{r}_{1f}=\mathbf{r}_{2f}=\mathbf{r}}. \quad (3)$$

In the stationary limit the result does not depend on time. Accordingly we can suppress the dependence of σ on the time t_f and set $t_i \rightarrow -\infty$. We also assume the current to be averaged over the final position $\mathbf{r}_{1f} = \mathbf{r}_{2f} = \mathbf{r}$. Hence, the conductivity (3) does not depend on this coordinate.

When the initial density matrix is chosen to be the equilibrium form ρ_0 of non-interacting electrons – which does not evolve in time – the expression for the conductivity can be simplified further. Since Eq. (3) contains no time dependence for times earlier than t' , the path integral can be restricted to times later than t' . Hence we get

$$\sigma = \frac{e^2}{3m} \int_{-\infty}^{t_f} dt' \int d\mathbf{r}'_1 d\mathbf{r}'_2 [\nabla_{\mathbf{r}_{1f}} - \nabla_{\mathbf{r}_{2f}}] J_0(t_f, t', \mathbf{r}_{1f}, \mathbf{r}_{2f}, \mathbf{r}'_1, \mathbf{r}'_2) \times [\mathbf{r}'_1 - \mathbf{r}'_2] \rho_0(\mathbf{r}'_1, \mathbf{r}'_2) \Big|_{\mathbf{r}_{1f}=\mathbf{r}_{2f}}, \quad (4)$$

where the time evolution operator J_0 is obtained from Eq. (2) by setting $V_x = 0$. The expression (4) can be shown to be equivalent to the standard Kubo formula for non-interacting electrons (see e.g. Ref. 20).

In metals, where the electron wavelength is short, one can evaluate the path integral J_0 in the ‘quasiclassical’ approximation. This assumes that the main contributions arise from classical paths $\mathbf{r}^{\text{cl}}(t)$ and quadratic fluctuations around them. In the present problem we have to specify the paths on both branches of the ‘Keldysh contour’, $\mathbf{r}_1^{\text{cl}}(t)$ and $\mathbf{r}_2^{\text{cl}}(t)$. The main contributions to the conductivity (4) arise from pairs of identical classical paths, $\mathbf{r}_1^{\text{cl}}(t) = \mathbf{r}_2^{\text{cl}}(t)$. They yield the Drude conductivity $\sigma_D = 2e^2 N_0 D$, where D is the diffusion constant, proportional to the elastic mean free time τ_e . As shown by Altshuler, Aronov and Khmelnitsky (AAK)³ and emphasized by Chakravarty and Schmid (CS)⁴, the first quantum correction to the conductivity, due to ‘weak localization’, is derived from pairs of time-

Low-temperature Dephasing ...

reversed classical paths $\mathbf{r}_2^{\text{cl}}(t' + t) = \mathbf{r}_1^{\text{cl}}(t_f - t)$. They yield the correction

$$\delta\sigma_{\text{WL}}^0 = -\frac{2e^2 D}{\pi} \int_{\tau_e}^{\infty} dt W(t), \quad (5)$$

where $W(t) = (4\pi Dt)^{-d/2} a^{d-3}$ is the classical probability for a diffusive particle to return to an initial point after time t . Here d is the dimensionality of the sample and a the film thickness for $d = 2$, or the square root of the wire cross section (area) for $d = 1$. The weak localization correction diverges in one and two dimensions due to the contributions from the upper limit of the integration. This divergence is removed, when interactions are taken into account. Their effect is to cut the integral at times exceeding the ‘dephasing time’ τ_φ . This time should be determined in the following.

Under ideal situations at low temperatures the dephasing time τ_φ is limited by electron-electron interactions. In order to determine it we proceed in two stages. We first present a hand-waving approach in the spirit of AAK³ and CS⁴, which is accepted to be sufficient at not too low temperatures. In the next section we will substantiate and improve the derivation.

The essence of the simple approach is to replace the interaction by an equivalent Nyquist noise source. I.e. the electrons move in the effective fluctuating Gaussian potential $\delta V(t, \mathbf{r})$ with the correlator

$$\langle \delta V(t, \mathbf{r}) \delta V(0, 0) \rangle \equiv I(t, \mathbf{r}) = \int \frac{d\omega d^3 k}{(2\pi)^4} \text{Im} \left(\frac{-4\pi \coth(\omega/2T)}{k^2 \epsilon(\omega, k)} \right) e^{-i\omega t + i\mathbf{k}\mathbf{r}}, \quad (6)$$

which is fixed by the fluctuation-dissipation theorem. Here $\epsilon(\omega, k)$ is the dielectric susceptibility of the material. The fluctuating potential should be added⁴ to the action of the electron, $S_0 \rightarrow S = \int_{t_i}^{t_f} dt [m\dot{\mathbf{r}}^2/2 - U(\mathbf{r}) + e\delta V(t, \mathbf{r})]$, which enters, e.g., the expression for the conductivity (3).

In the presence of the Nyquist noise a reduction of the time span of the path integration, as done in the transition from (3) to (4), is no longer rigorously allowed since the density matrix now depends on time. Any arbitrary density matrix at $t_i = -\infty$ should relax to the equilibrium density matrix of the interacting problem. The use of a reduced form equivalent to (4) with, e.g., the equilibrium density matrix of the non-interacting problem at time t' , in general may lead to spurious results describing the decay of this specific initial state. However, as will be shown below, for the weak localization correction this approximation is justified with sufficient accuracy.

Due to the Gaussian nature of the fluctuating field δV one can perform the average and finds that the exponent in Eq. (3) acquires an imaginary part,

$$e^{iS_0[\mathbf{r}_1] - iS_0[\mathbf{r}_2] - S_1[\mathbf{r}_1, \mathbf{r}_2]}, \quad (7)$$

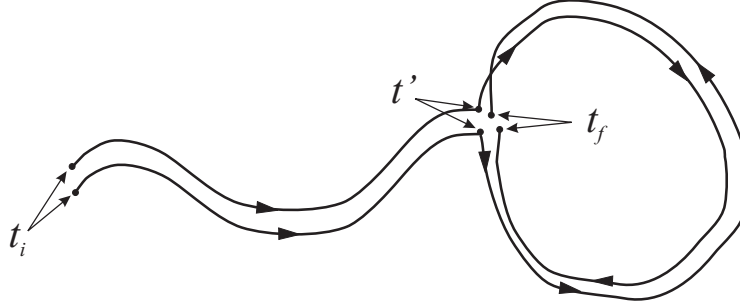


Fig. 1. A pair of classical paths which contributes to the weak localization correction.

with

$$S_I = \frac{e^2}{2} \int_{t_i}^{t_f} dt \int_{t_i}^{t_f} d\tilde{t} [I(t - \tilde{t}, \mathbf{r}_1(t) - \mathbf{r}_1(\tilde{t})) + I(t - \tilde{t}, \mathbf{r}_2(t) - \mathbf{r}_2(\tilde{t})) - I(t - \tilde{t}, \mathbf{r}_1(t) - \mathbf{r}_2(\tilde{t})) - I(t - \tilde{t}, \mathbf{r}_2(t) - \mathbf{r}_1(\tilde{t}))]. \quad (8)$$

The effects of interactions, which are responsible for the decay of correlations and coherence, are lumped in the imaginary part of the action S_I . As long as the interactions are weak we can analyze their effect within the quasiclassical approximation. For this we need to evaluate S_I on the important classical paths $\mathbf{r}_2(t)$ and $\mathbf{r}_1(t)$, satisfying the classical equation of motion without interaction $\delta S_0[\mathbf{r}]/\delta \mathbf{r} = 0$. Since we are interested in weak localization corrections, we concentrate on paths involving pairwise time-reversed paths. The leading contribution arises from pairs of paths as shown in Fig. 1. The two paths are equal, $\mathbf{r}_2(t) = \mathbf{r}_1(t)$, for times earlier than t' , they are time-reversed, $\mathbf{r}_2(t) = \mathbf{r}_1(t_f + t' - t)$, for times $t' < t < t_f$, and they pass the same region within a diameter of order of the mean free path l at two times, $t = t'$ and $t = t_f$. In principle, there could be additional loops with time-reversed parts before t' . However, we can ignore such contributions because they constitute higher-order weak localization corrections to the conductivity, which decay faster in time. For the paths considered, it is easy to see that the double time integral in Eq. (8) reduces to

$$\int_{t_i}^{t_f} dt \int_{t_i}^{t_f} d\tilde{t} \rightarrow \int_{t'}^{t_f} dt \int_{t'}^{t_f} d\tilde{t}. \quad (9)$$

The imaginary part of the action $S_I(t_f - t')$ grows with the time difference $t_f - t'$. For a given pair of paths it may fluctuate on a short time scale of order τ_e , however this averages out when we sum over all possible paths.

Low-temperature Dephasing ...

The classical paths of electrons in disordered metals are diffusive. When we add the contributions of all possible pairs of time-reversed paths a decaying factor $\langle W(t_f - t') e^{-S_I(t_f - t')} \rangle$ appears under the integral (4). Our aim is to find the characteristic time scale τ_φ characterizing this decay. In order to do that, we may replace $\langle W(t) e^{-S_I(t)} \rangle$ by $W(t) e^{-\langle S_I(t) \rangle}$. This simple averaging has been also performed by CS⁴, while AAK³ evaluated directly $\langle W(t) e^{-S_I(t)} \rangle$. However, both approaches gave the same expression for τ_φ up to a prefactor of order one, which is of little importance.

After these steps the weak localization correction to the conductivity (5) is modified to become

$$\delta\sigma_{\text{WL}} = -\frac{2e^2 D}{\pi} \int_{\tau_e}^{+\infty} dt W(t) e^{-f_d(t)}, \quad (10)$$

where $f_d(t) = \langle S_I(t) \rangle$ has the form

$$f_d(t) = e^2 \int_0^t dt' \int_0^t dt'' \int d^d x D(|t' - t''|, x) [I(t' - t'', x) - I(t' + t'' - t, x)]. \quad (11)$$

It depends on the ‘diffuson’ $D(t, r) = (4\pi Dt)^{-d/2} a^{d-3} \exp(-r^2/2t)$, i.e. the probability that an electron propagates the distance r within a time t .

Using the Drude formula for the dielectric susceptibility of disordered metal, $\epsilon(\omega, k) = 4\pi/(-i\omega + Dk^2)$, we can express the function $f_d(t)$ as¹⁸

$$\begin{aligned} f_d(t) = & \frac{\pi^{\frac{3d-d^2-2}{2}} e^2 (4D)^{1-\frac{d}{2}}}{2^{3d-d^2-2} \sigma_D a^{3-d}} \int \frac{d\omega d\omega'}{(2\pi)^2} |\omega'|^{\frac{d}{2}-2} \left[(\omega - \omega') \coth \frac{\omega - \omega'}{2T} \right. \\ & \times \left. \frac{1 - \cos \omega t}{\omega^2} - \omega \coth \frac{\omega}{2T} \frac{\cos \omega t - \cos \omega' t}{\omega'^2 - \omega^2} \right]. \end{aligned} \quad (12)$$

It is obvious that $f_d(t)$ grows with time and limits the range of time integration. The dephasing time can be defined by the condition $f_d(\tau_\varphi) \sim 1$, with the result¹⁰

$$\frac{1}{\tau_\varphi} = \frac{4e^2 D}{\sigma_D a^{3-d}} \int_{1/\tau_\varphi}^{1/\tau_e} \frac{d\omega}{2\pi} \int \frac{d^d q}{(2\pi)^d} \frac{\omega \coth(\omega/2T)}{\omega^2 + (Dq^2)^2}. \quad (13)$$

Explicitly we find^{10,17}

$$\begin{aligned} 1/\tau_\varphi &= \frac{e^2}{\pi \sigma_D a^2} \sqrt{\frac{2D}{\tau_e}} [1 + 2T \sqrt{\tau_\varphi \tau_e}] & \text{in 1D,} \\ 1/\tau_\varphi &= \frac{e^2}{4\pi \sigma_D a \tau_e} [1 + 2T \tau_e \ln(T \tau_e)] & \text{in 2D,} \\ 1/\tau_\varphi &= \frac{e^2}{3\pi^2 \sigma_D \sqrt{2D} \tau_e^{3/2}} [1 + 6(T \tau_e)^{3/2}] & \text{in 3D.} \end{aligned} \quad (14)$$

The divergence of the integral in (13) at high frequencies is cut at $\omega \sim 1/\tau_e$ because a classical path needs a time exceeding τ_e to return to the same point. Because of the quantitative ambiguity related to this cut-off procedure, as well as the definition of $1/\tau_\varphi$, the results are valid only up to numerical prefactors of order one.

We observe that the dephasing rate (13) does not vanish even at zero temperature, and, in this respect, differs from the earlier expectations. The main difference between our work and that of AAK³ and CS⁴ lies in the fact that they ignore fluctuations of the potential with frequencies higher than T . This procedure yields a dephasing rate which vanishes at $T = 0$. For a comparison of the results it is useful to split (13) as

$$\frac{1}{\tau_\varphi(T)} = \frac{1}{\tau_{\varphi 0}} + \frac{1}{\tau(T)}. \quad (15)$$

Here $\tau_{\varphi 0}$ is our – controversial – zero temperature dephasing time, and $\tau(T)$ is basically the result derived by AAK³. The latter has also been supported by Aleiner, Altshuler and Gershenson (AAG)²¹. Within the framework of first order perturbation theory in the interaction they arrive at

$$\frac{1}{\tau_\varphi} = \frac{e^2 D}{T \sigma_D a^{3-d}} \int_{1/\tau_\varphi}^{\infty} \frac{d\omega}{2\pi} \int \frac{d^d q}{(2\pi)^d} d\xi \frac{\omega [\coth(\omega/2T) + \tanh((\xi - \omega)/2T)]}{(\omega^2 + (Dq^2)^2) \cosh^2(\xi/2T)}. \quad (16)$$

In this form, after performing the integration over ξ one observes that high frequencies $\omega > T$ do not contribute to the integral, and (16) effectively reduces to the result obtained by AAK³. Below we will comment on the arguments for either of the results. Before doing so we present, in the following section, a more thorough analysis of interaction effects.

3. INTERACTION EFFECTS: MORE RIGOROUS APPROACH

In this Section we review a more thorough description of interaction effects in disordered conductors, based on the work of Refs. 17 and 18, which takes into account Pauli principle etc. The formulation of the problem of electron-electron interaction in metals is similar in many respects to the problem of a particle interacting with a bath of harmonic oscillators, which is known as the Caldeira-Leggett (CL) model.

First, the fluctuating electro-magnetic field needs to be quantized. This is achieved by introducing two independent fluctuating potentials V_1 and V_2 for the two branches of the Keldysh contour (see, e.g., Ref. 22) or, equivalently, the combinations $V^+ = (V_1 + V_2)/2$ and $V^- = V_1 - V_2$. We assume

Low-temperature Dephasing ...

the fluctuations to be small, and expand the action up to the second order in V^\pm . This approximation is equivalent to RPA. The first order term in the fields describes the motion of electrons in a effective static potential of other electrons. It leads to the so-called Hartree diagrams in perturbation theory with closed electronic loops. In a homogeneous Jellium model these terms vanish because of charge neutrality, but in real disordered metals this is not the case. Nevertheless, these terms are not important for dephasing, and we ignore them, leaving in the action only the terms quadratic in V^\pm . Then the properties of the electro-magnetic field are fully determined by the correlators of V^\pm , i.e. the photon Green functions. In homogeneous cases they have the form:

$$\langle V^+(t, \mathbf{r}) V^+(0, 0) \rangle = I(t, \mathbf{r}), \quad \langle V^+(t, \mathbf{r}) V^-(0, 0) \rangle = iR(t, \mathbf{r}), \quad \langle V^- V^- \rangle \equiv 0, \quad (17)$$

where

$$R(t, \mathbf{r}) = \int \frac{d\omega d^3k}{(2\pi)^4} \frac{4\pi}{k^2 \epsilon(\omega, k)} e^{-i\omega t + i\mathbf{k}\mathbf{r}}, \quad (18)$$

and the function $I(t, \mathbf{r})$ is defined by Eq. (6). Note that the function $R(t, \mathbf{r})$ vanishes identically for $t < 0$ due to causality.

The density matrix of an interacting electron in a metal, $\rho(\mathbf{r}_1, \mathbf{r}_2) = \langle \Psi^+(\mathbf{r}_2) \Psi(\mathbf{r}_1) \rangle$, can be expressed in terms of the density matrix $\rho_V(\mathbf{r}_1, \mathbf{r}_2)$ of an electron in the fluctuating potentials V^\pm as follows:

$$\rho(\mathbf{r}_1, \mathbf{r}_2) = \langle \rho_V(\mathbf{r}_1, \mathbf{r}_2) \rangle_{V^\pm}. \quad (19)$$

The averaging over V^\pm is performed with the aid of (17). The matrix $\rho_V(\mathbf{r}_1, \mathbf{r}_2)$ satisfies the generalized nonlinear Liouville equation:

$$i \frac{\partial \rho_V}{\partial t} = [H_0 - eV^+, \rho_V] - \frac{1}{2} (1 - \rho_V) eV^- \rho_V - \frac{1}{2} \rho_V eV^- (1 - \rho_V). \quad (20)$$

Here and below in Eq. (21-26) the products are understood in an operator sense and integrations over internal spatial (not time) coordinates are assumed. Eq. (20) is exact, accounting, for instance, for the Pauli principle. The properties of the interaction are determined by the correlators (17). Eq. (20) had been derived in Ref. 17, and below we will outline this derivation. We can add that Eq. (20) has been independently re-derived by Eriksen *et al.*²³ and also by von Delft¹⁹, who applied Wick's theorem.

The full 2×2 matrix Keldysh Greens function $\check{G}_V(t_1, t_2, \mathbf{r}_1, \mathbf{r}_2)$ in the fluctuating potentials $\check{V} = V_i \delta_{ij}$ (for $i = 1, 2$) satisfies the equation

$$(i\check{1}\partial/\partial t_1 - \check{1}H_0 + e\check{V})\check{G}_V = \check{\sigma}_z \delta(t_1 - t_2) \delta(\mathbf{r}_1 - \mathbf{r}_2). \quad (21)$$

D.S. Golubev, A.D. Zaikin, and G. Schön

It follows from Eq. (21) that the matrix elements of \check{G}_V can be expressed by the evolution operators $U_i(t_1, t_2) = \mathbf{T} \exp \left[-i \int_{t_1}^{t_2} dt (H_0 - eV_i) \right]$ as follows:

$$\begin{aligned} G_{11,V} &= -i\theta(t_1 - t_2)U_1(t_1, t_2) + iU_1(t_1, t)\rho_V(t)U_1(t, t_2), \\ G_{22,V} &= -i\theta(t_2 - t_1)U_2(t_1, t_2) + iU_2(t_1, t)\rho_V(t)U_2(t, t_2), \\ G_{12,V} &= iU_1(t_1, t)\rho_V(t)U_2(t, t_2), \\ G_{21,V} &= -iU_2(t_1, t)[1 - \rho_V(t)]U_1(t, t_2). \end{aligned} \quad (22)$$

On the other hand, \check{G} satisfies the Dyson equation

$$\check{G}_V(t_1, t_2) = \check{G}_0(t_1, t_2) - \int_{t_i}^t dt' \check{G}_0(t_1, t') e\check{V}(t') \check{G}_V(t', t_2), \quad (23)$$

where the Keldysh matrix \check{G}_0 is defined by Eq. (22) with $V_i = 0$. Eq. (22) is compatible with Eq. (23) only if

$$\rho_V(t) = U_1(t, t_i) \{1 - U_1(t, t_i) \rho_0 [U_1(t_i, t) - U_2(t_i, t)]\}^{-1} \rho_0 U_2(t_i, t), \quad (24)$$

where ρ_0 is an arbitrary time-independent initial density matrix. One can verify directly that the density matrix (24) satisfies the equation of motion (20). This completes the derivation of (20).

The solution (24) in principle allows one to derive an exact expression for the conductivity. Alternatively, we derive here the conductivity directly from (20). In the presence of a weak external electric potential, $V_x(\mathbf{r}) = -\mathbf{E}\mathbf{r}$, Eq. (20) can be linearized in V_x to yield

$$i \frac{\partial \delta \rho_V}{\partial t} = H_1 \delta \rho_V - \delta \rho_V H_2 - [eV_x, \rho_V], \quad (25)$$

where $\delta \rho_V$ is proportional to V_x and

$$H_1 = H_0 - eV^+ - \frac{1}{2}(1 - 2\rho_V)eV^-, \quad H_2 = H_0 - eV^+ + \frac{1}{2}eV^-(1 - 2\rho_V). \quad (26)$$

Continuing along the lines outlined in Section 2, we find the conductivity,

$$\begin{aligned} \sigma &= \frac{e^2}{3m} \int_{-\infty}^{t_f} dt' \int d\mathbf{r}'_1 d\mathbf{r}'_2 [\nabla_{r_{1f}} - \nabla_{r_{2f}}] \\ &\quad \times \left\langle J_V(t_f, t', \mathbf{r}_{1f}, \mathbf{r}_{2f}, \mathbf{r}'_1, \mathbf{r}'_2) [\mathbf{r}'_1 - \mathbf{r}'_2] \rho_V(t', \mathbf{r}'_1, \mathbf{r}'_2) \right\rangle_{V^\pm} \Big|_{\mathbf{r}_{1f}=\mathbf{r}_{2f}}, \end{aligned} \quad (27)$$

where

$$J_V(t, t', \mathbf{r}_{1f}, \mathbf{r}_{2f}, \mathbf{r}'_1, \mathbf{r}'_2) = \int \mathcal{D}\mathbf{p}_1 \mathcal{D}\mathbf{r}_1 \mathcal{D}\mathbf{p}_2 \mathcal{D}\mathbf{r}_2 e^{iS_1[\mathbf{p}_1, \mathbf{r}_1, V^\pm] - iS_2[\mathbf{p}_2, \mathbf{r}_2, V^\pm]}, \quad (28)$$

Low-temperature Dephasing ...

and

$$S_{1,2} = \int_{t'}^{t_f} dt \left[\mathbf{p}\dot{\mathbf{r}} - \frac{\mathbf{p}^2}{2m} - U(\mathbf{r}) + eV^+(t, \mathbf{r}) \pm \frac{[1 - 2\rho_V(\mathbf{p}, \mathbf{r})]}{2} eV^-(t, \mathbf{r}) \right]. \quad (29)$$

The only approximation used so far is RPA, which allows a restriction to Gaussian fluctuations of the potentials V^\pm . If one averages over V^\pm with the aid of full nonlinear action of the electro-magnetic field, then Eqs. (27-29) are exact. It is also useful to note at this point that the form of the action (29) is not unique since the Hamiltonians (26) are sensitive to the ordering of momentum and coordinate operators. Under these circumstances details of quantization procedure become important. However, all these uncertainties are of quantum origin and disappear in the quasiclassical limit $\hbar \rightarrow 0$, which will be the only interesting limit in what follows.

From here on we resort to further approximations. *First*, in Eq. (27) we replace ρ_V by the equilibrium density matrix of non-interacting electrons ρ_0 . As has been pointed out in Ref. 19 this is equivalent to the assumption of a factorized initial density matrix in the CL and similar models, an approximation which is sufficient in many cases, although for certain correlators it is known to fail in the long-time limit¹². *Second*, also in Eq. (29) we replace ρ_V by ρ_0 . It is possible to show that for the problem of weak localization both these approximations are justified. We will return to this point in the discussion in Section 4. Here we only like to note that this statement can be justified by rewriting the exact solution (24) in the form of a power series, each term of which can be handled without replacing ρ_V by ρ_0 .

Having made these approximations, we perform the averaging over V^\pm in Eq. (7) and get the conductivity:

$$\sigma = \frac{e^2}{3m} \int_{-\infty}^{t_f} dt' \int d\mathbf{r}'_1 d\mathbf{r}'_2 [\nabla_{\mathbf{r}_{1f}} - \nabla_{\mathbf{r}_{2f}}] J(t_f, t', \mathbf{r}_{1f}, \mathbf{r}_{2f}, \mathbf{r}'_1, \mathbf{r}'_2) \Big|_{\mathbf{r}_{1f}=\mathbf{r}_{2f}} \times [\mathbf{r}'_1 - \mathbf{r}'_2] \rho_0(\mathbf{r}'_1, \mathbf{r}'_2), \quad (30)$$

where

$$J(t_f, t', \mathbf{r}_{1f}, \mathbf{r}_{2f}, \mathbf{r}'_1, \mathbf{r}'_2) = \int \mathcal{D}\mathbf{p}_1 \mathcal{D}\mathbf{r}_1 \int \mathcal{D}\mathbf{p}_2 \mathcal{D}\mathbf{r}_2 \times e^{i\tilde{S}_0[\mathbf{p}_1, \mathbf{r}_1] - i\tilde{S}_0[\mathbf{p}_2, \mathbf{r}_2] - iS_R[\mathbf{p}_1, \mathbf{r}_1, \mathbf{p}_2, \mathbf{r}_2] - S_I[\mathbf{r}_1, \mathbf{r}_2]}, \quad (31)$$

and $\tilde{S}_0[\mathbf{p}, \mathbf{r}] = \int_{t'}^{t_f} dt [\mathbf{p}\dot{\mathbf{r}} - \mathbf{p}^2/2m - U(\mathbf{r})]$. The parts of the action iS_R and S_I describe the effect of the bath of all electrons on the motion of a single electron. We do not give explicit expressions for S_R , since it vanishes for

time-reversed diffusive paths (see Ref. 17 for details) and thus is irrelevant for dephasing²⁴. Here we would only note that this part of the action contains the combination $1 - 2\rho_0(\mathbf{p}, \mathbf{r}) = \tanh[(H_0(\mathbf{p}, \mathbf{r}) - \mu)/2T]$, which is responsible for the Pauli exclusion principle and generates the term with $\tanh[(\xi - \omega)/2T]$ in first order perturbation theory (16). The imaginary part of the action S_I is given by Eq. (8). It is positive for all possible pairs of paths \mathbf{r}_1 and \mathbf{r}_2 , except for two identical ones when it is zero. Only S_I contributes to dephasing. In the previous section we have shown that it leads to the decay of correlations $\propto \exp(-f_d(t))$, where $f_d(t)$ is given by Eq. (12).

The derivation of Eq. (30) does not involve disorder averaging, except that we have used a translationally invariant form of the voltage correlators (18), which is appropriate after averaging. On the other hand, one can demonstrate that the result (30) remains valid in the general case when the correlators depend on two arguments separately. We also like to point out that the formula (30) is very similar to the evolution equation for the density matrix of a particle interacting with the bath of oscillators (CL).

4. DISCUSSION

The derivation presented above leads to the conclusion that the dephasing time of electrons in disordered metals, as measured in weak localization experiments, remains finite at low temperatures. This result deviates from the earlier theoretical understanding and accordingly has been heavily criticized. In this section we summarize several of the arguments presented against this conclusion and our counterarguments. This discussion should further clarify under which circumstances the present result applies.

First-order perturbation theory suggests vanishing $T = 0$ dephasing.

Based on a first-order perturbation expansion AAG²¹ concluded that the dephasing rate is given by expression (16). It contains the combination $\coth(\omega/2T) + \tanh[(\xi - \omega)/2T]$, which vanishes in the limit $\xi, T \rightarrow 0$.

In contrast, the result (13) contains only $\coth(\omega/2T)$ and remains finite for $T \rightarrow 0$. The term $\tanh[(\xi - \omega)/2T]$ in (16) comes from the real part of the action S_R , which vanishes for time-reversed pairs of classical paths, and thus is of no importance for the decay of coherence in the long time limit.

Perturbation theory probes only short times. In general it cannot be used for the calculation of τ_φ , since this would require an extrapolation to long times, e.g., by exponentiation of the first order perturbative result. Such a step is ambiguous, as illustrated by the well-studied problem of the

Low-temperature Dephasing ...

quantum decay of a metastable state in the presence of dissipation. The semi-classical decay rate is $\Gamma = B \exp(-A)$. The exponent $A = A_0 + \eta A_1$ is the action on a saddle-point path¹¹. Here the friction coefficient η describes the interaction with a dissipative bath, and $A_{0,1}$ (as well as $B_{0,1}$ below) are positive and independent of η . Fluctuations around the saddle-point path contribute to the pre-exponent B . In the weak damping limit one gets¹² $B \simeq B_0 + \eta B_1$. It is obvious that this result and the specific separation into prefactor and exponent – although it can be expanded in η – cannot be derived unambiguously from a purely perturbative calculation. Similarly, for the calculation of the dephasing rate it is important to separate the pre-exponent, which is of little importance, from the contributions to the exponent. In contrast to the first order calculation the saddle-point analysis of GZ^{10,17} provides a definite prescription for this separation.

When comparing saddle-point and first-order calculation one should note that the former concentrates on contributions from time-reversed pairs of classical paths whereas the latter effectively sums up contributions from all paths. Therefore, after the saddle-point approximation is performed one cannot recover any more certain details of the first-order calculation.

We, furthermore, would like to point out that even in the framework of perturbation theory, after exponentiation one would find a non-vanishing – though small – dephasing rate at $T = 0$. It is not contained in Eq. (16) which is based on Fermi's golden rule. However, already first order perturbation theory in the interaction yields additional non-golden-rule terms^{21,18}. These terms arise due to the non-Markovian nature of the interaction, as a result of which the usual Golden-rule replacement of $[1 - \cos(\omega t)]/\omega^2$ by a δ -function fails at low temperature. The extra contributions are usually small compared to the Golden-rule terms, but they are the leading ones when the combination of coth and tanh terms cancel. These extra terms also contribute to dephasing at $T = 0$.

In the ground state there is no electron scattering.

This argument is based on the known properties of inelastic scattering. As long as the energy of an incident particle is lower than that of the first excited state of the scatterer, inelastic scattering is forbidden. The internal ground state wave function of the scatterer manifests itself only through a form- or Debye-Waller factor. One may conclude then that at low temperatures all electrons are in the ground state, therefore electron-electron scattering is suppressed and no dephasing occurs.

This argument is not conclusive for several reasons. First, if one would describe electron-electron interaction in terms of inelastic scattering one should take into account that the scatterer is a macroscopic interacting many

particle system with a very small spacing between the energy eigenstates. The temperatures where the experiments have been performed are much higher than this level spacing.

Second, since the Coulomb interaction is long-range each electron interacts with many other ones. A perturbative electron-electron scattering picture is actually not appropriate. A more appropriate description is in terms of collective electron degrees of freedom, for example plasmons. Also the Nyquist noise description used here accounts for collective properties. Within this picture all electrons form the environment for a given electron.

Quasiparticles are fully coherent.

First we recall that the existence of exact quasiparticles is not proven, especially in disordered metal. Even if well-defined quasiparticles do exist, it would not rule out dephasing of real electrons. It is the electric current of real electrons which is measured. An electron can be understood as a combination of an *infinite* number of coherent quasiparticles (provided the latter can be constructed). Therefore it may be only partially coherent even at $T = 0$. A similar scenario can be analyzed rigorously in a Caldeira-Leggett-type model, where one (singled-out) oscillator is coupled to an infinite bath of oscillators. The single oscillator loses phase coherence even though the eigenmodes of the total system are fully coherent.

During disorder averaging Hikami boxes were missed.

AAG²¹ argued that careful disorder averaging within perturbation theory introduces certain contributions, the ‘Hikami boxes’. According to them, Hikami boxes correspond to some paths in our approach, which we have missed. This would be responsible for the finite dephasing at $T = 0$.

On one hand, we understand that Hikami boxes do not correspond to any classical path. They appear as a formal feature of the perturbative many-body approach used by AAG. There Hikami boxes are needed to restore the causality principle. In the path integral approach causality is always maintained, and Hikami boxes, together with unusual paths, do not appear at all. Moreover, even within the approach of AAG Hikami boxes appear only in the terms generated by the action S_R , which are irrelevant for dephasing.

On the other hand, within our approach the dephasing rate is found to be practically insensitive to the particular averaging procedure employed. For any given impurity configuration we find that for *all* time-reversed paths the term $S_I(t)$ is positive and grows with time. Thus for *any* pair of such paths quantum interference is suppressed by a factor $\sim \exp(-S_I(t))$. Different procedures of averaging over these paths or the impurity configurations, e.g. $\langle W(t)e^{-S_I(t)} \rangle$ or $W(t)e^{-\langle S_I(t) \rangle}$, should yield essentially the same value of

Low-temperature Dephasing ...

dephasing rate, possibly differing by prefactors of order one.

The replacement $\rho_V \rightarrow \rho_0$ is uncontrolled.

In Ref. 19 it was argued that the replacement $\rho_V \rightarrow \rho_0$ in Eq. (27) is dangerous. This replacement can be interpreted as preparation of an artificial initial state of the system. It might lead to spurious relaxation processes, which could be interpreted as dephasing at $T = 0$.

When analyzing this question it is important to note that we do not study the time evolution of the density matrix of an electron, rather the conductivity of the electron system. Both evolve differently as can be seen from the comparison of Eqs. (1) and (4). The initial density matrix $\rho_0(\mathbf{r}_{1i}, \mathbf{r}_{2i})$ in Eq. (1) is replaced by the combination $(\mathbf{r}'_1 - \mathbf{r}'_2)\rho_0(\mathbf{r}'_1, \mathbf{r}'_2)$ in Eq. (4). The latter can be interpreted as an effective density matrix, which is strongly non-equilibrium due to the presence of the factor $(\mathbf{r}'_1 - \mathbf{r}'_2)$. This ‘non-equilibrium’ form is the main reason of relaxation and dephasing, rather than the assumption of factorized initial conditions. More generally, one observes that in dissipative systems most of the correlators of the type $\langle \hat{A}(t)\hat{A}(0) - \hat{A}(0)^2 \rangle$ decay with time even at $T = 0$, which does not contradict to the fact that the equilibrium density matrix does not relax.

In spite of these arguments, one might still worry that the replacement $\rho_V \rightarrow \rho_0$ could lead to an incorrect long-time asymptotic and wrong conclusions about the magnitude of τ_φ . This indeed happens for certain correlators in exactly solvable models¹². We find, however, that it is not the case for the conductivity in the weak localization problem. We have already shown in Sec. 2 that τ_φ is not sensitive to the replacement if we can ignore the field V^- . The latter can be ignored since it is responsible only for the appearance of the part S_R in the action, which gives zero contribution to $1/\tau_\varphi$. We can further add that even if we include V^- , the replacement $\rho_V \rightarrow \rho_0$ is justified. The reason is that the integration in Eq. (27) is confined to the region $|\mathbf{r}'_1 - \mathbf{r}'_2| < l$. In this range the effect of interactions on the density matrix is negligible because l is much shorter than the dephasing length L_φ . Finally, we have developed an alternative approach based on a rewriting of the exact solution (24) in the form of a series. Within this approach the replacements $\rho_V \rightarrow \rho_0$ are not made, however the final result for τ_φ remains unchanged.

High frequency modes can be ignored at $T = 0$.

The difference between the present and earlier results for the dephasing rate lies to a large extent in the inclusion or truncation of high frequency modes. Their relative role can be demonstrated if we consider a model problem of a particle interacting with a bath of harmonic oscillators. An example of such a system is a tunnel junction with high resistance in a

D.S. Golubev, A.D. Zaikin, and G. Schön

circuit involving a dissipative environment. The properties of such a system are described by the so-called $P(E)$ -theory²⁶. The function $P(E)$ is the Fourier transform of the self-correlation function of the operator $\exp(i\hat{\varphi})$, with $\varphi = \int_{t_0}^t dt' eV(t')$ being the phase of the junction:

$$P(E) = \frac{1}{2\pi} \int dt e^{J(t)+iEt}, \quad e^{J(t)} = \langle e^{i\hat{\varphi}(t)} e^{-i\hat{\varphi}(0)} \rangle. \quad (32)$$

The function $J(t)$ can be found exactly, $J(t) = -F(t) - iK(t)$, where

$$F(t) = \frac{e^2}{\pi} \int_0^\infty d\omega \operatorname{Re}[Z(\omega)] \omega \coth \frac{\omega}{2T} \frac{1 - \cos \omega t}{\omega^2}, \quad (33)$$

$$K(t) = \frac{e^2}{\pi} \int_0^\infty d\omega \operatorname{Re}[Z(\omega)] \frac{\sin \omega t}{\omega}. \quad (34)$$

Here $Z(\omega)$ is the total impedance of the system tunnel junction plus environment. Note that the function $F(t)$ is similar, although not identical to the function $f_d(t)$ introduced in (12). It is also possible to show that $F(t)$ comes from the imaginary part of the action of the environment S_I , while $K(t)$ corresponds to the real part S_R .

We can rewrite the function $F(t)$ approximately as a sum of two terms,

$$F(t) \approx 2T \frac{e^2}{\pi} \int_0^T d\omega \operatorname{Re}[Z(\omega)] \frac{1 - \cos \omega t}{\omega^2} + \frac{e^2}{\pi} \int_0^\infty d\omega \operatorname{Re}[Z(\omega)] \frac{1 - \cos \omega t}{\omega}.$$

AAK and other authors considered only the first, ‘thermal’ term in the theory of weak localization. The second term is ‘quantum’. It contains the contribution of high frequencies.

We first consider the properties of the function $F(t)$ at $T = 0$, which depend strongly on the impedance $Z(\omega)$. If $\operatorname{Re}[Z(\omega)] = 0$ below certain frequency ω_{\min} , then $\exp[-F(t)]$ first decays with some characteristic time τ_φ , approaches a constant at time $t \sim 1/\omega_{\min}$, and beyond this time the correlations do not decay any more. Now the role of different frequencies on the process of dephasing becomes clear. The high frequency properties of the impedance determine the initial stage of decay and introduce the characteristic decay time τ_φ , while the low frequency modes determine the long-time behavior of the correlations. If the low frequency modes are absent, then the decay of correlations stops at time $t \sim 1/\omega_{\min}$.

Finite temperature do not change this picture qualitatively. The decay becomes stronger than at $T = 0$. At $t > 1/\omega_{\min}$ the decay of correlations stops again, but the asymptotic value of the correlation function $\langle e^{i\hat{\varphi}(t)} e^{-i\hat{\varphi}(0)} \rangle$ is smaller than for $T = 0$.

Low-temperature Dephasing ...

On the other hand, if the function $\text{Re}[Z(\omega)]$ is non-zero at small ω , the correlations decay completely in the long time limit at any temperature including $T = 0$.

Further arguments.

Since the publication of Refs. 10,17 many more arguments have been put forward challenging the conclusions reached in these articles. To list a few of them:

- The Pauli principle is not taken into account properly.
- The electric field is not quantized.
- The effect of interaction on classical paths is ignored.
- Detailed balance is violated.
- The term iS_R cancels S_I for $T \rightarrow 0$.

Here we only want to stress that we have checked these as well as other arguments and found in all cases that they did not apply or are in error.

5. EXPERIMENT

In our earlier work we have already demonstrated that our theory fits the data of Ref. 6. Here we want to add a comparison with some more recent experiments. Natelson *et al.*⁹ found that their data for τ_φ are not consistent with the geometry dependence predicted by the standard theory³, and for wide samples they observed saturation of τ_φ at low temperatures. In the work of the Saclay-Michigan collaboration no saturation was detected in silver²⁷ and pure gold²⁸ samples down to ≈ 50 mK, but a clear saturation was observed in copper at temperatures one order of magnitude higher. We will show that these results can be explained within the framework of quantum dephasing due to electron-electron interactions.

First we note that Eqs. (14) are valid only in strictly 1D, 2D or 3D geometries. In practice, however, the diffusion depends on the geometry of a particular sample and intermediate regimes may occur. For instance in the experiments of Refs. 9,27,28 the samples are quasi-1d (quasi-2d) in a standard sense, i.e. both $L_\varphi = \sqrt{D\tau_\varphi}$ and $L_T = \sqrt{D/T}$ exceed the thickness t and width w of the sample. However, the elastic mean free path l is smaller than t and w . Therefore the diffusion is three-dimensional at short times. Since the main contribution to the dephasing at $T = 0$ comes from high frequencies, the 3d expression

$$\frac{1}{\tau_{\varphi 0}^{(3d)}} = \frac{e^2}{3\pi^2 \sigma_D \sqrt{2D}} \left(\frac{b}{\tau_e} \right)^{3/2} \quad (35)$$

applies. (The cut-off dependent numerical factor b is of order 1.) On the other hand, the temperature-dependent part of the rate, see Eq. (15), comes from low frequencies or long times, when the diffusion is one-dimensional, and the corresponding form should be used.

In Ref. 9 the maximum dephasing times for ten (6 quasi-1d and 4 quasi-2d) samples fabricated from the same material (*AuPd*) with practically the same resistivity were found to be nearly universal $\tau_\varphi^{max} \approx (0.8 \dots 2) \times 10^{-11}$ sec, independent of the sample geometry. In the Saclay-Michigan experiments^{27,28} the maximum dephasing times were found to be 2 to 3 orders of magnitude longer than those in Ref. 9. Furthermore, τ_φ^{max} for the silver sample was about an order of magnitude higher than that for the copper sample in spite of similar (though not identical) parameters²⁷. Observing this difference the authors²⁷ suggested that the low temperature saturation of τ_φ in disordered metal wires is material dependent and not universal.

For our comparison we will use the data for 8 samples of Ref. 9 with nominally identical resistivity $\rho \approx 24 \mu\Omega \text{ cm}$ and $D \approx 1.5 \times 10^{-3} \text{ m}^2/\text{sec}$ (samples C to F and H to K), 2 samples of Ref. 27 (*Ag* and *Cu*) and one gold sample (*Au*) of Ref. 28 (denoted there by *AuMSU*). Since for all samples of Ref. 9 the maximum dephasing times were nearly the same, we only quote an average τ_φ^{max} . Experimental values of τ_φ^{max} and our theoretical predictions for $\tau_{\varphi 0}$ (Eq. (35) with $b = 1$) are summarized in the following Table:

sample	τ_φ^{max} (sec)	$\tau_{\varphi 0}$ (sec)
C to F and H to K (averaged) ⁹	1.3×10^{-11}	0.3×10^{-11}
<i>Ag</i> ²⁷	10×10^{-9}	2×10^{-9}
<i>Cu</i> ²⁷	1.8×10^{-9}	0.3×10^{-9}
<i>Au</i> ²⁸	8×10^{-9}	4×10^{-9}

We note that for all the above samples the expression (35) with $b = 1$ gives correct order-of-magnitude estimates for τ_φ^{max} , even though these times differ by 2 to 3 orders of magnitude. By adjusting the cut-off parameter in the range $b \approx 0.3 \dots 0.6$ we could produce a quantitative agreement with all the mentioned experimental values. Alternatively we can eliminate the ambiguity related to the cutoff b by considering the ratios between the maximum dephasing times for different samples. One finds:

	experiment	theory (eq. (35))
$\tau_{\varphi 0}^{AuPd} / \tau_{\varphi 0}^{Ag}$	1.3×10^{-3}	1.5×10^{-3}
$\tau_{\varphi 0}^{AuPd} / \tau_{\varphi 0}^{Cu}$	0.7×10^{-2}	1×10^{-2}
$\tau_{\varphi 0}^{AuPd} / \tau_{\varphi 0}^{Au}$	1.6×10^{-3}	0.8×10^{-3}

This agreement strongly supports the conclusion about the universality of the low temperature saturation of τ_φ in disordered conductors. To com-

Low-temperature Dephasing ...

plete this summary we mention that also our $1d$ and $2d$ expressions for the maximum dephasing time (14) give correct order-of-magnitude estimates for τ_φ^{max} for the above samples because the mean free path is comparable to the width and thickness of the wires.

The comparison with the standard theory³ is much worse. The authors of Ref. 9 found that the value of τ_φ which follows from AAK for their $2d$ samples differs already at 1K significantly from their experimental values. At the lowest temperature, $T \approx 80$ mK, this would imply a deviation by 3 to 4 orders of magnitude. Only for the thinnest samples the observed temperature dependence was consistent with $\tau_\varphi \propto T^{-2/3}$. However, in these cases the values of the dephasing time were such that $T\tau_\varphi < 1$ for the whole temperature range, whereas for the standard theory³ $T\tau_\varphi \gg 1$ is required.

We now analyze the temperature dependence of the dephasing time τ_φ detected in the experiments of Refs. 9 and 27,28. For quasi-1d samples Eq. (15) can be written in the following dimensionless form¹⁰

$$\frac{\tau_{\varphi 0}}{\tau_\varphi} = 1 + \frac{T}{T_0} \sqrt{\frac{\tau_\varphi}{\tau_{\varphi 0}}}, \text{ where } T_0 = \frac{R_K \sigma_D t w}{4\tau_{\varphi 0}^{3/2} \sqrt{2D}}. \quad (36)$$

Here $R_K = h/e^2 \simeq 25.8$ k Ω is the resistance quantum. For a comparison with

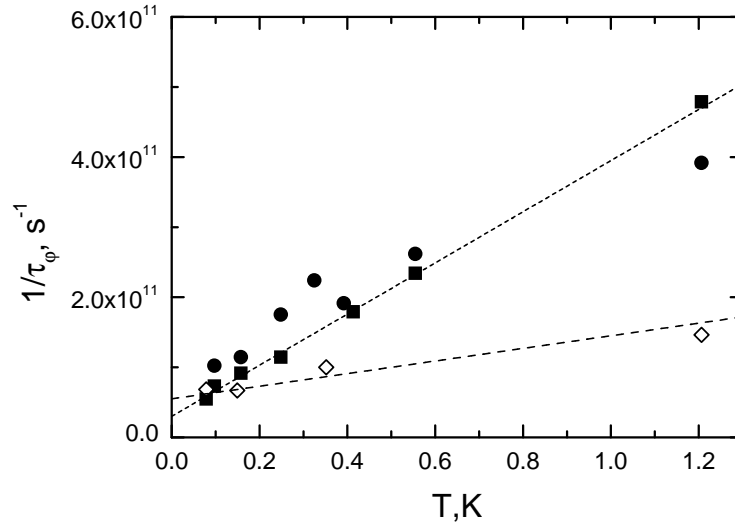


Fig. 2. Dephasing rate $1/\tau_\varphi$ measured in Ref. 9 for the samples A (squares), B (circles) and F (open symbols) as a function of temperature T . The straight lines are guides to the eye.

the power-law temperature dependence predicted by AAK the experimental

data had been presented on a double-logarithmic plot. However, since the temperature $T_0 \sim 10\text{K}$ is high and the dependence of Eq. (36) is roughly linear, it is more useful to re-plot them on a linear scale. For samples with similar parameters ρ , D and $\tau_{\varphi 0}$ the slope for quasi-1d samples should depend only on the cross section tw . In particular, since w for the sample F of Ref. 9 is reported to be 4 times larger than for the samples A to E, one expects the slope to differ by a factor 4. The experimental data for the samples A, B and F, presented in Fig. 1, are in good agreement with these expectations, with a ratio 4.05 between the slopes corresponding to the samples A and F. On the other hand, the magnitude of the slopes turns out to be larger than predicted by Eq. (35,36). The difference is as large as a factor of order 8 when $b = 1$, reducing to a factor $3 \dots 4$ when the cut-off parameter is chosen in the range $b = 0.3 \dots 0.5$. This agreement is acceptable within the accuracy of our theory. We conclude that our theory accounts for the main observations of Ref. 9.

Now we will address the results of Refs. 27,28. In Fig. 3 of Ref. 27 the authors presented their data for *Au*, *Ag* and *Cu* samples. These three samples had similar classical parameters, but both the magnitude and the temperature dependence of τ_{φ} differ drastically. This observation led the authors to conclude that the behavior of τ_{φ} and, in particular, its saturation at low temperature may be material dependent. Later it was demonstrated by the same group²⁸ that the unusually low values of τ_{φ} in gold²⁷ were most likely due to high concentration of magnetic impurities. A pure gold sample was fabricated and a similar behavior was observed²⁸ as had been found previously²⁷ for the *Ag* sample: τ_{φ} showed no saturation on a log-log plot down to $T \sim 50$ mK. At the same time it was confirmed²⁸ that *Cu* samples with very high purity showed saturation similarly to the earlier *Cu* sample²⁷.

A material dependence of τ_{φ} cannot be ruled out in general. However, we find that such a dependence is not needed in order to quantitatively explain the seemingly different behavior of *Ag* (no saturation down to $T \sim 50$ mK) and *Cu* (clear saturation already at $T \gtrsim 700$ mK). The ‘saturation’ temperature T_0 , defined in (36), sets the scale at which thermal and quantum contributions to the dephasing rate become comparable. It is obvious that for otherwise identical parameters the thinner samples will show saturation at lower temperatures. For the parameters of the *Cu*²⁷, *Ag*²⁷ and *Au*²⁸ samples and with the cutoff parameter $b = 1$ one finds from Eq. (35,36) $T_0^{Cu} \approx 2.8$ K, $T_0^{Ag} \approx 90$ mK and $T_0^{Au} \approx 40$ mK. It is important to note that, for example, the ratio $T_0^{Cu}/T_0^{Ag} \sim 30$ is not sensitive to ambiguities related to the cutoff parameter b . We conclude that the temperature where quantum effects set in can easily vary by more than an order of magnitude for samples with similar macroscopic parameters.

Low-temperature Dephasing ...

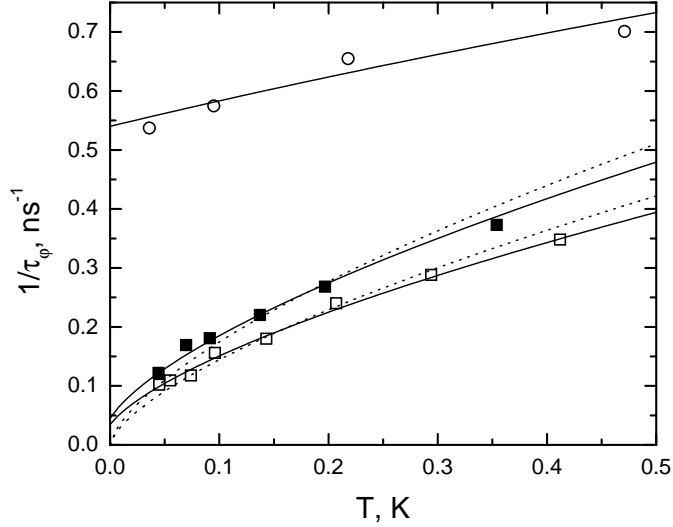


Fig. 3. The values of $\tau_\varphi(T)$ for the samples *Cu* (circles), *Ag* (open squares) of Ref. 27 and *Au* (black squares) of Ref. 28. Dotted lines are the best fits to the formula³ $1/\tau_\varphi = AT^{2/3}$. Solid lines are the best fits to Eq. (36) with both $\tau_{\varphi 0}$ and T_0 varied.

In Fig. 3 we replot the data points for three samples of Refs. 27,28 on a linear scale. Best fits to the Eq. (36) are shown by solid lines, while dotted lines correspond to the standard theory³. Obviously the saturating *Cu*-data can be fitted only with a non-vanishing dephasing rate at $T = 0$. Also the ‘non-saturating’ samples *Ag* and *Au* can be fitted with a finite $\tau_{\varphi 0}$, although one cannot draw definite conclusions since the temperatures T_0 are too low. From the fits we find $T_0^{Cu} = 1.2$ K, $T_0^{Ag} = 14$ mK, $T_0^{Au} = 16$ mK and $\tau_{\varphi 0}^{Cu} = 2$ ns, $\tau_{\varphi 0}^{Ag} = 30$ ns, $\tau_{\varphi 0}^{Au} = 20$ ns. The deviations from the values given above can again be reduced if we adjust the cut-off parameter in the range $b = 0.3 \dots 0.7$. We conclude that the experimental data of Refs. 27,28 are consistent with the predicted low-temperature saturation of the dephasing. No material dependence of τ_φ needs to be assumed to get this agreement.

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